Lola’s Calculus Travels!

Meet Lola! She is a spunky girl who loves mathematics. Her parents are world travelers, and they decided to take some time off to travel the world. Lola is super excited, but also sad because she couldn’t study calculus at school. Being the good Samaritan that you are, you offer to teach Lola calculus as she travels so she can continue her beloved math studies!



Lola is on board with this super cool idea! She wants to master one calculus topic during each visit to a different country! She refuses to leave to country until she has learned all of the material successfully,. Each visit to a country takes one day, but if you get a question wrong on the “Lola tries” problems, you must stay an extra day! Your challenge is to be back to North Carolina in 120 days, the earlier the better, in time for Lola’s cousin, Dora the Explorer’s birthday!



Lola loves to play hide and seek, and so she will be hidden in certain places throughout the manual! See how many times you can find her. Yes, you can count the Lola on this page as the first time you see her! The amount of times Lola is in the whole manual will be posted at the end of the Answer Key at the back!

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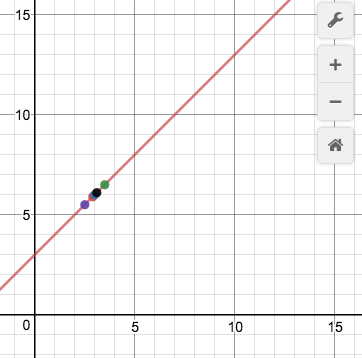
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# Welcome to North Carolina!

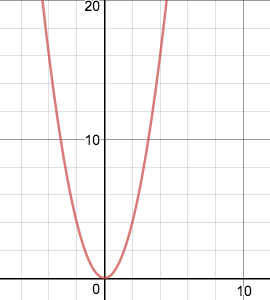
What is a limit?

**The limit (L) of a function, f(x), as “x” approaches “a” refers to the behavior of the function as x-values get closer and closer to the values “a”. We do not care what actually happens at x=a, just the behavior of the graph as we get closer and closer to it. It is mathematically written as . This limit only exists if and . L would be your y value. This basically means that the limit from the left (a-) must equal the limit from the right (a+) for the limit to exist. Let’s investigate this idea further!

 This is the graph of the function, . Let’s see what happens to the behavior of the function as we approach the x-value of 3 on this function from the left. We can look at a point somewhat close to x= 3 from the left, at x=2.5. In this case, the coordinate would be (2.5,5.5). But, we can do even better! What about at x=2.9? The coordinate for this point would be (2.9, 5.9). We can choose x values closer and closer to 3, without ever actually reaching it. In fact, there are an infinite number of numbers that get awfully close to the coordinate at x=3, (3,6), without ever touching it. This helps us determine the behavior of the graph as we get closer and closer to x=3 from the left. The y coordinates for x=2.5 and x=2.9 are getting closer and closer to a y value of 6, which would mean that the

Great! We are halfway done. To find the overall limit, we must also look from at the behavior as x approaches 3 from the right. Let’s choose a number that is a bit larger than 3, x=3.5. The coordinate would be at (3.5, 6.5). If we chose an x-value a little closer, perhaps at x=3.1, our coordinate would be (3.1, 6.1). Again, as we get closer and closer to x=3, our y value is getting closer to 6. This means that the . Since the limit from the left equals the limit from the right, we know that the . Again, we do not care what the actual coordinate is at x=3, just what the behavior of f(x) is from either side of x=3.

## Guided Practice

1. What does mean? What does mean? Does the overall limit exist?
   1. As x-values get closer and closer to the 1 from the left, the function f(x) is getting closer and closer to 2. As x-values get closer and closer to 1 from the right, the function f(x) is getting closer and closer to 3. The overall limit does not exist because the limit from the left does not equal the limit from the right!
2. The function is shown on the right. What is the ?
   1. To determine the limit, we would look at the behavior of the graph as we choose x-values closer and closer to x=0. First, we would have to look at the limit from the left. If we chose an x-value a bit smaller than 0, perhaps x=-.1, the y-value would be .01. If we chose an x-value to the left of x=0, but a little bit closer, perhaps x=-.001, the y value would be .00001. The y-values are getting closer and closer to 0, so . Now, we must look at it from the right. If we chose an x-value a bit larger than 0, perhaps x=.1, the y-value would be .01. If we chose an x-value still to the right of x=0, but a little bit closer, perhaps x=.001, the y-value would be .000001. The y-values are getting closer and closer to 0, so . Since the limit from the left equals the limit from the right, we know that .

## ../Desktop/Screen%20Shot%202016-11-10%20at%2010.50.25%20AM.pngLola Tries

1. What does mean?
2. Does the exist if and ?
3. What is the ? The graph of g(x) is shown on the right.

**Next Stop: Mexico**

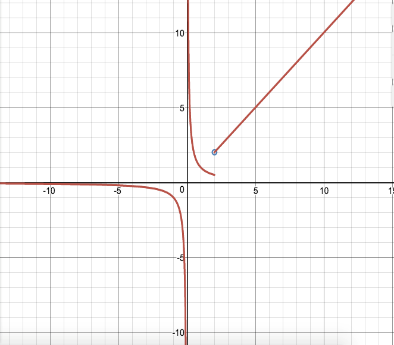
# ../Desktop/mexico2.jpgWelcome to Mexico! *../Desktop/mexico1.jpg*

Bienviendo a Mexico!

How do I evaluate limits from a graph and a table?

Now that we understand what a limit is, it’s time to start looking at how to evaluate limits using graphs and tables. As a reminder, a limit is the behavior of a function as x approaches “a”. With this in mind, we are ready to start!

|  |  |
| --- | --- |
| x | f(x) |
| .9 | 1.09 |
| .99 | 1.99 |
| 1.1 | 2.01 |
| 1.19 | 2.1 |

What is the Well, to evaluate the limit, we must look at the behavior from both sides of x= 1. As x-values get closer and closer to 1 from the left, the y-values are getting closer to 2. The same is true from the right! So we know that the because of this. That’s pretty easy right? Well, let’s look at a harder example. ****

|  |  |
| --- | --- |
| x | f(x) |
| 2.5 | 2.9 |
| 2.9 | 2.999 |
| 3 | 5 |
| 3.5 | 2.999 |
| 3.9 | 2.9 |

What is the At first glance, you would probably think that this limit evaluates to 5. Think again! Yes, f(x)=5, but the the . This is because a limit doesn’t care about the behavior at a point, but rather the behavior around a point. As x-values get closer and closer to 3 from the left, f(x) is also getting closer and closer to 3. As x-values get closer and closer to 3 from the right, f(x) is also getting closer and closer to 3. Since the limit from the left equals the limit from the right (both of them being 3), the overall limit also equals 3.

We can use this same process when evaluating limits from graphs. Given the graph of to the left, what is the

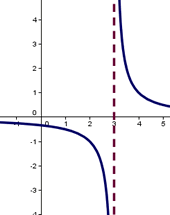
Looking at the graph, we see that on the left side, we see that the “j” shaped part of the curve approaches a y-value of 0.5 Looking at the right side of x = 2, we see that the linear portion of the graph *approaches* a y-value of 2, even if there is a hole at x = 2. Based upon this, we know that does not exist, because the limits from both sides are not equal.

## Guided Practice

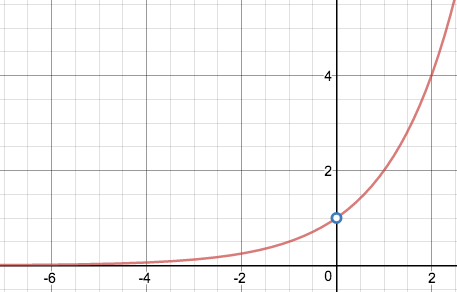
1. What is the ?

|  |  |
| --- | --- |
| x | f(x) |
| 5 | 6.1 |
| 6 | 6.5 |
| 7 | 6.9 |

* 1. To determine what is we must look at the behavior of f(x) as it gets closer and closer to x=6 from both sides. In this case, we know that f(x)= 6.5, and that the function is linear in this portion because as the x values increase by 1, the y values increase by a constant amount, 0.4. With this information, we know that = f(x)= 6.5 because f(x) is getting closer and closer to 6.5 from both the left and the right of x=6.

1. What is the ?
   1. To determine what is we must look at the behavior of f(x) as it gets closer and closer to x=3 from both sides. From the right, the graph is continuously increasing. There is not one specific value for f(x) at x=3. Since the graph is continuously increasing in the positive direction, we can say that the . The limit as x approaches 3 from the right is positive infinity. Now we must look at the behavior from the left side. From the left of x=3, the graph is continuously decreasing. Since it is continuously decreasing in the negative direction, we can say that the . The limit as x approaches 3 from the left is negative infinity. Since the limit from the left does not equal the limit from the right, , the limit as x approaches 3 does not exist. ****

## Lola Tries

1. On the graph to the right, what is ?
2. On the graph to the right, what is ?
3. On the table below, what is the?

|  |  |
| --- | --- |
| x | f(x) |
| 1.9 | 4.9 |
| 1.99 | 4.99 |
| 2 | 17 |
| 2.1 | 4.99 |

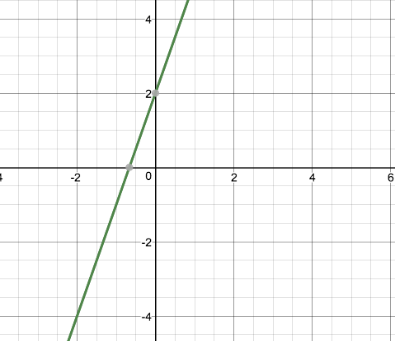
Next Stop: Costa Rica

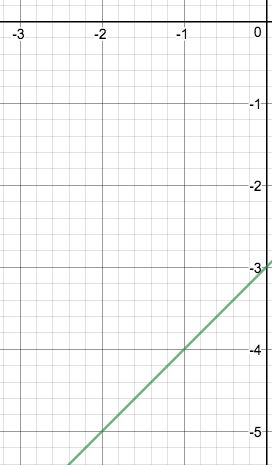
# Welcome to Costa Rica!

Bienviendo a Costa Rica!

How do I evaluate limits algebraically? 

We have the basic skills and understanding to move into the next major part of evaluating limits: how to evaluate limits algebraically. All of previous concepts we have learned are still in play when we evaluate limits algebraically, we are just adding one more tool for evaluation.

 To find the limit for any continuous function, we use substitution. For example, if we have the function and we wanted to know the , we would simply substitute x=-2 into f(x). Therefore, . We can see that this is in fact the case when we view the graph of f(x). If we evaluate the limit from the graph of f(x), we can see that the is indeed, -4. There are two major exceptions to the substitution rule. The first exception is when substitution yields , where c is any constant. We will get to this exception in a later section!

 The second exception is when substitution yields the indeterminate form, . In this case, we will need to change the form of the function using various algebraic techniques in order to evaluate it. For example, if we had the function , and if we wanted to know the , we would first use substitution: . We now must use algebraic techniques to manipulate the function into something that we can work with. Since the top is a quadratic function, a good first step may be to factor it. . We can change the numerator to this, so that we now have . Since there is a (x+2) on both the top and the bottom, we can cancel, so that we have . Finally, we can substitute -2 in for x to evaluate the limit. . We can see from the graph of the function that .

## Guided Practice

1. What is the ?
   1. The first thing one should try doing is substitution. With substitution we yield . Since we got the indeterminate form, we know we must use some algebraic manipulation to successfully solve the limit. The first thing to do is factor the quadratic in the denominator of the second fraction. We would then have . Now, we can try to combine the two fractions into one. We need to have a common denominator. The common denominator can be (x)(x+1). The first fraction is missing an x+1, so we would multiply the numerator and denominator of the first fraction by x+1 to get . Since we have an x on both the numerator and denominator, we can cancel them to get . We have algebraically manipulated the limit so that we are finally able to use substitution. Now we can substitute 0 for x to get . So the .
2. What is the ?
   1. The first thing one should try is substitution. With substitution we yield = 0. Since substitution worked, we are done! =0!

## Lola Tries

1. What is the ?
2. What is the ?
3. What is the

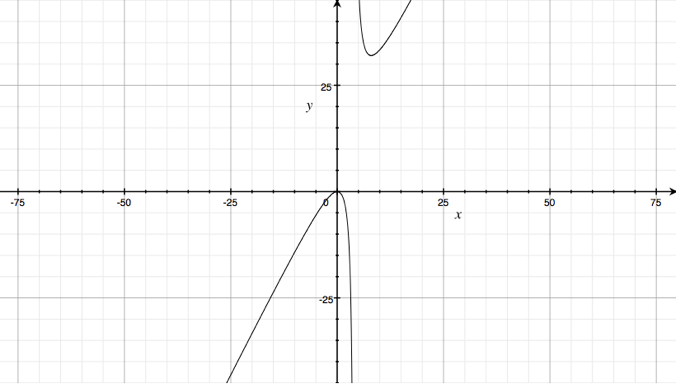
Next Stop: Panama

# Welcome to Panama!

La bienvenida a Panamá!

How do I use limits to determine vertical asymptotes? 

****

 The first exception to using substitution is when substitution yields , where c is any constant. Any limit resulting in will have a limit equal to or DNE. For example, if we had a function, and we wanted to know the , we would first try to use substitution. . Since substitution did not work, we would look at the limit from either side of x=4. To evaluate the limit from the left of x=4, we would plug in a number that is a bit smaller than x=4. We would indicate this by . We would then see if the numerator is positive or negative if we plugged in a number a bit smaller than 4. Then we would see if the denominator is positive or negative if we plugged in a number a bit smaller than 4. Lastly, we would see if the overall limit from the left is positive or negative. If the limit is negative, then the . If the limit is positive, then the .Now we are ready to evaluate the limit from the left: . When we substituted a number smaller than -4, we yielded a positive numerator and negative denominator. Since a positive divided by a negative is a negative, we know that the . Great! We are now halfway done. In order to evaluate the overall limit, we must look at the limit from both sides. We will now look at the limit as x approaches 4 from the right. To evaluate the limit from the left of x=4, we would plug in a number that is a bit larger than x=4. We would indicate this by . We would then follow the same steps that we did when evaluating the limit from the left. The limit as x approaches 4 from the right: . Now that we know the limit from both the left and the right side, we can determine the overall limit. Since the limit as x approaches -4 from the left is and the limit as x approaches -4 from the right is , the overall limit does not exist, as the limit from the left does not match the limit from the right. If we were to evaluate the limit from the graph of f(x) as we did in the last section, we would see that .

## Guided Practice

1. What is the ?
   1. The first thing one should try doing is substitution. With substitution we yield . Since we got a constant over 0, we know that the limit will evaluate to or DNE. This is an indication that we must look at the limit from both sides. The limit from the left: The limit from the right: . Since the limit from the left does not equal the limit from the right, the overall limit does not exist. So
2. What is the ?
   1. The first thing one should try doing is substitution. With substitution we yield . Since we get a constant over 0, we know the limit will evaluate to or DNE. This is an indication that we must look at the limit from both sides. The limit from the left . The limit from the right: . Since the limit from the left does not equal the limit from the right, the overall limit does not exist. So

## Lola Tries

1. What is the ?
2. What is the ?
3. What is the ?

Next Stop: Brazil

# Welcome to Brazil!

Bem vindo ao Brazil!

What are limit laws?

There are laws for limits?! Absolutely! What are they? Limit laws are essentially shortcuts to evaluate certain limits. The first four limit laws have to do with limits of two functions. Let’s dive right in!

1. Suppose that c is a constant and the limits and exist, then
   * If we are to take the limit of f(x) plus g(x) approaching “a”, then we are able to evaluate the limit as f(x) approaches “a” and g(x) approaches “a”, separately, and simply add them together.
   * If we are to take the limit of f(x) minus g(x) approaching “a”, then we are able to evaluate the limit as f(x) approaches “a” and g(x) approaches “a”, separately, and simply subtract them.
   * If we are to take the limit of f(x) times g(x) approaching “a”, then we are able to evaluate the limit as f(x) approaches “a” and g(x) approaches “a”, separately, and simply multiply them together.
   * If we are to take the limit of f(x) times g(x) approaching “a”, then we are able to evaluate the limit as f(x) approaches “a” and g(x) approaches “a”, separately, and simply divide them.

The next seven limit laws deal with other limit situations, but we keep the assumption that c is a constant.

* + If we are to take the limit of f(x) as it approaches “a”, but f(x) is multiplied by some constant, we can take the constant out and evaluate the limit of f(x) as it approaches “a” normally, but multiply the limit by the constant at the end.
  + If we are to take the limit of f(x) as it approaches “a”, but it is raised to the nth power, we can simply take the limit of f(x) as it approaches “a” normally, and then raise our result to the nth power. In this situation, it is important to remember that n must be a positive integer.
  + The limit of any constant as it approaches “a” is just the value of that constant.
  + The limit of the function, y=x, as it approaches “a”, is just the value of “a”, the value that the function is approaching.
  + The limit of the function, x raised to the nth power, as it approaches “a”, is just “a” raised to the nth power. Again, we must remember that n will have to be a positive integer.
  + The limit of the function, the nth root of x, as it approaches “a”, is just the nth root of “a”. Not only does n have to be a positive integer, if it is even, we must assume that a>0.
  + If we are to find the limit of the function, the nth root of f(x), as it approaches “a”, we find the limit as f(x) approaches “a” like normal, and then take the nth root of our result.

## Guided Practice

1. What is ?
   1. The first thing we can do is identify that this would be a situation in which we could use limit laws, even though it may not look like it! We can define the numerator as f(x), and the denominator as g(x). Now we know we can use limit law #4 to evaluate this situation. We first take the limit of the numerator. We can simply use substitution in this situation. = 21. Next we take the limit of the denominator, using substitution. . We can determine the overall limit now. We take the limit of the numerator (21) and divide it by the limit of the denominator (22) to get the overall limit as 21/22. So, !
2. If we know that the limit of f(x) as it approaches 2 is 5 and the limit of g(x) as it approaches 2 is 2, what is ?
   1. In this situation, there are multiple limit laws going on! We know that when we are taking the limit of two functions added together, we can look at them as separate limits and then just add them together according to limit law #1. So let’s focus on the first function, f(x). Limit law #6 tells us that we can simply take the limit of f(x) as it approaches 2 first, then square it. We know that the limit of f(x) as it approaches 2 is 5, and if we square that it is 25. Great! We can now look at the g(x) part. In this situation, x is multiplied by g(x) squared. x is a constant because it is just an x-value. In this situation, x would be 2 because we are approaching 2 for our limit. Using limit law #5, we know we can simply multiply this constant by the limit of g(x) squared as it approaches 2. Since g(x) is squared, we must use limit law #6 again and take the limit of g(x) as it approaches 2 separately and then raise it to the second power. The limit of g(x) as it approaches 2 is 2, and if we square that it is 4. We must remember to also multiply that by the constant, 2. So the limit for that section of the overall limit is 8. When we take the limit of two functions added together, we simply add the component limits. The first section gave us 25 and the second section gave us 8, so 25+8 = 31. So, .

## Lola Tries

1. Let and . What is ?
2. What is the ?
3. What is the ?

Next Stop: Peru

# Welcome to Peru!

La bienvenida a Perú!

How do I use limits to determine horizontal asymptotes?

Before we can use limits to determine horizontal asymptotes, we must first understand what a horizontal asymptote is! A horizontal asymptote exists if or , where L is just a constant. On the graph to the right, , a horizontal asymptote exists at y=0, because as the x values get closer and closer to infinity and negative infinity, the y-value gets closer and closer to 0, as the denominator is getting smaller and smaller.

Now we are ready to use limits! There is an important theorem to remember when using limits to determine horizontal asymptotes. If r>0 and r is a rational number, then and . This can be especially useful in determining limits to infinity for fractions, as we can multiply the numerator and denominator by 1 over x raised to the highest power, and a lot of the components of the limit will go to zero.

For example, if we wanted to evaluate the following limit; , we would start by identifying the highest power. The highest power in this limit is 2. So we would multiply both the numerator and denominator by . We can start by dividing each component of the polynomial in the numerator by . , , as there is a higher power on the denominator than the numerator, and , for the same reasoning as the previous component. So on the numerator, we are left with only a 2. We would do this same process on the denominator. , , and . So on the denominator, we are left with only 8. We are left with the fraction . Therefore, . We can see this by examining the graph to the left. As the x values get larger and larger towards infinity, the y values are getting closer and closer to ½ .

## Guided Practice

1. What is the ?
   1. First, we must identify the highest power in the whole fraction. The highest power is Therefore, we would multiply both the numerator and denominator by When multiplying it to the numerator, we must multiply it by , as it’s being multiplied under square root, and . We would multiply to every component of the polynomial under the square root. In doing so, we would only be left with the on the numerator, because is a higher power than , or a constant, so those two limits would go to zero, and , but we have to take the square root of 4, which is 2. Then, we would multiply to the denominator, to get just 30 on the denominator, because is a higher power than just x, and so that limit would go to zero, and . Thus, our overall limit would go to . Thus
2. What is ?
   1. Though this does not look like a fraction right now, we can easily make it one by multiplying by a conjugate on both the numerator and denominator. In this case, the denominator is 1. Once we multiply by a conjugate () and simplify on the numerator, we are left with . The highest power in this instance would be . We must multiply to both the numerator and denominator. When we multiply it to the numerator, we get . On the denominator, we must multiply the inside of the radical by , as . Once we do so, we get a from the piece of the denominator inside of the radical, as is a higher power than x, and so that limit goes to zero, and , but we have to take the square root of 25, which is 5. Lastly, we take care of the 5x that is added outside of the radical. We multiply this by to get 5, as . So we are left with an 8 on the numerator and a 5+5=10 on the denominator, so our overall limit evaluates to . Thus, .

## Lola Tries

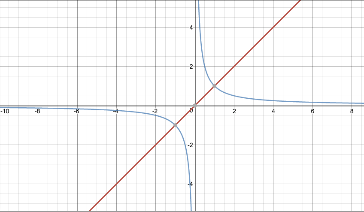
1. What is ?
2. What is ?
3. What is ?

Next Stop: Argentina

# Welcome to Argentina!

La bienvenida a la Argentina!

How can I use limits to prove continuity at a point?

A function is continuous when . In other words, a function is continuous at a given point, “a”, when the left-hand limit is equal to the right-hand limit and is equal to the value of the function at this point. For instance, the function , graphed in red on the right, is continuous, as there are no “breaks” in the function. The function , graphed in blue, is not at x=0 continuous, as there is a “break” at this point and the graph is separated into two different parts. There is a three-step proof in order to prove continuity at a point:

1. f(a) is defined
   * The original point must first exist before we can prove continuity! We know that f(a) is defined if “a” is in the function, f’s, domain. If f(a) did not exist, the right hand part of our definition for continuity () would not work.
2. exists
   * We must then prove that the limit exists. In order to prove that the limit as f(x) approaches “a” exists if the limit from the left side equals the limit from the right side. If did not exist, the left hand part of our definition for continuity () would not work.
3. * We must check if the value of f(a) from step #1 equals the limit as f(x) approaches “a”. If so, we have proven that f(x) is continuous at x=a.

For example, if , is f(x) continuous at x=-2? First, we must check if f(-2) exists, and what that value is. We know to use the top function because that piece of the function is defined when x is greater than or equal to -2. Thus, plugging in x=-2, we find that .

Now we must check if the limit exists, and if so, what the value is! The limit as x approaches -2 from the right is -4. The first piece of the piecewise function is when x is greater than or equal to 2, so we use this value for the limit as well; therefore, . Since the limit from the left equals the limit from the right, the overall limit exists at -4.

Since , both yield a y value of -4, f(x) is continuous at x=-2!

## Guided Practice

1. If , is g(x) continuous at x=1
   1. First, we would find the value of g(x) at x=1, using the top part of the piecewise function. Using substitution, we find that g(1)= -2
   2. We know the limit as g(x) approaches 1 from the right is -2, as we would substitute 1 into the top part of the piecewise function as we did in part a. Now we must find the limit from the left, substituting 1 into the bottom part of the piecewise function. . Since the limit from the left does not equal the limit from the right, we can stop there because the right hand side of the definition of continuity is not met. Therefore, g(x) is not continuous at x=1.
2. For , find a value of c to make f continuous at x=2.
   1. To solve this, we must understand that in order for continuity to occur, the limit from the left and the right must match and f(a) must exist. For the limits to match however, . Using this piece of knowledge, we can solve for c. We know x=2, so . Thus, c must equal 5/2 for f(x) to be continuous at x=2.

## Lola Tries

1. If ; is f(x) continuous at x=4?
2. Find the values of a and b to make the function continuous:
3. If is f(x) continuous at x=2?

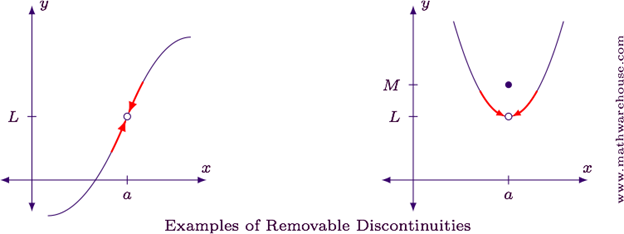
Next Stop: South Africa

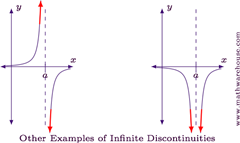
# Welcome to South Africa!

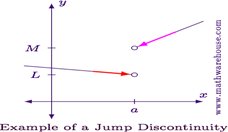
Ukuwamukela eNingizimu Afrikha!

What are the different types of discontinuity and how do I identify them?

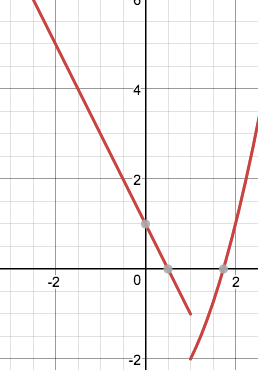
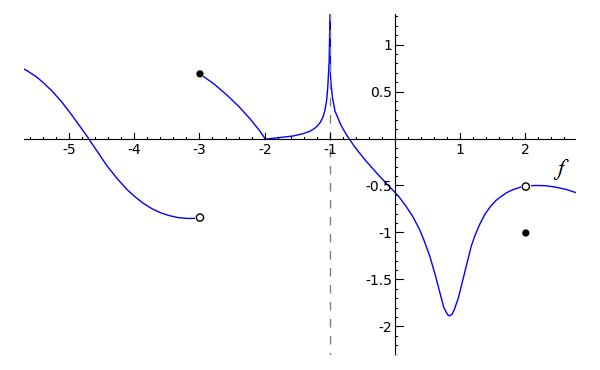
Now that we know when a function is continuous, it is important to explore when and how a function is discontinuous! There are three major ways that a function can be discontinuous.

The first type of discontinuity is removable discontinuity. Removable discontinuity occurs at holes. In the example shown on the right, there is a hole on both graphs at x=a. In these cases, the limit as x approaches “a” always exists, but it’s discontinuous because f(a) either doesn’t exist, as shown on the graph to the left, or the limit as x approaches “a” does not match the point at x=a, as shown on the graph to the right! A hole exists when the limit as x approaches “a” upon substitution yields .

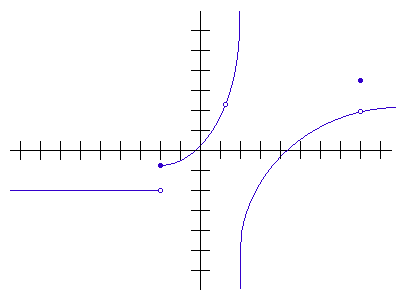
The second type of discontinuity is infinite discontinuity. Infinite discontinuity occurs where there is a vertical asymptote. A limit may or may not exist depending on the situation, but f(a) never exists. With infinite discontinuity, either the limit as x approaches “a” from the right, left, or both, must be . In the example to the left, there is infinite discontinuity in both graphs at x=a because a vertical asymptote exists and the limit as x approaches “a” for both graphs is . In the graph to the left, a limit does not exist because one side of the graph goes towards and the other side goes towards at x=a. In the graph to the right, a limit does exist because both sides of the graph are going towards . We know that infinite discontinuity exists when the limit as x approaches “a” upon substitution yields .

The last type of discontinuity is jump discontinuity. Jump discontinuity exists when the limit as x approaches “a” from the right does not equal the limit as x approaches “a” from the left. In other words, the limit as x approaches “a” does not exist. There may or may not be a point at x=a, depending on the situation. In the example shown to the right, jump discontinuity exists because the limit does not exist at x=a. Be wary though, do not get this confused with infinite discontinuity. Sometimes in infinite discontinuity, the limit does not exist either, however with infinite discontinuity, there is always a vertical asymptote.

## Guided Practice

1. Let’s revisit question #1 from our guided practice in the last section. We found in the last section that g(x) is discontinuous at x=1, but what kind of discontinuity is it?
   1. We found that the limit as x approaches 1 from the left does not equal the limit as x approaches 1 from the right. That eliminates removable discontinuity, because in removable discontinuity, the limit always exists. We are then left with jump discontinuity and infinite discontinuity. With infinite discontinuity, either the limit from the left or the limit from the right must be . The limit from the right was -2 and the limit from the left was -1, neither of which is . Using process off elimination, we can conclude that it is jump discontinuity.
2. Using the graph to the right, f(x), state the x values f(x) is discontinuous, the type of discontinuity, and explain why.
   1. x=-3, jump discontinuity, as the limit from the left does not equal the limit from the right, and neither the limit from the left nor from the right is approaching positive or negative infinity.
   2. x=1, infinite discontinuity, as the limit from both sides is approaching positive infinity.
   3. x=2, removable discontinuity, as f(2) does not match the limit at that point.

## Lola Tries

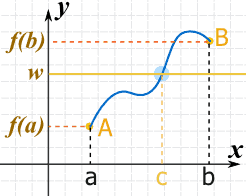
1. What type of discontinuity, if any, exists for the function, at x=0?
2. For the graph to the right, f(x), state the x-values at which f is discontinuous and the type of discontinuity.
3. For the function at what x values, if any does discontinuity occur, and what type?

Next Stop: Madagascar

# Welcome to Madagascar!

Welcome to Madagasikara!

What is the Intermediate Value Theorem?



The Intermediate Value Theorem tells us that if we are given a continuous function f defined on the closed interval [a,b], for any real number d between f(a) and f(b) there exists a point c between a and b such that f(c)=d. If we look at the graph on the right, we can see the Intermediate Value Theorem in action.

The graph shoes a continuous function, f. We know that the function is continuous because there are no “breaks” in the graph. We also have a closed interval [a,b], because the graph starts at a, and ends at b. and the point (a, f(a)) exists, and so does the point (b, f(b)). In between the x-values a and b, there is an x-value, c, in which f(c), which is defined as w on the graph, exists.

Imagine you and your friend want to stretch a single piece of rope from North Carolina to California. In order to do so, the rope has to travel through other states like Kansas, Nevada, etc. It can’t just magically appear in California, because it is a continuous piece of rope. Similarly, with the Intermediate Value Theorem, if you want to go from point a to point b, you must first pass through a point in the middle, point c.

The Intermediate Value Theorem is especially useful in finding whether a continuous function has a zero, or if a certain y-value exists on an interval.

For example, if we had the function , will the function have a zero on the interval ? We can use Intermediate Value Theorem to easily solve this problem. We know that the function is continuous, as both the sine and cosine functions are continuous. Then, we can find the y-values of our endpoints. , . We know that zero is between -1 and 1, and the graph is continuous, so there is a point on the graph in which f(c)=0! Therefore, by IVT, since f(0)<0<f() , there exists a “c” such that 0<c<, where f(c)=0. In other words, there is a place in the function, on the specified interval, where f(c)=0.

## Guided Practice

1. Determine if your oven is at 350°, as it cools down before turning it off, at some instant must its temperature be exactly 170°?
   1. We know that temperature is a continuous function, as it decreases slowly and fairly steadily. It will turn off at room temperature, which is approximately 70°. Temperature is a function of time. We can define time “0” as when the oven begins to cool off. Time is infinite, so the interval begins at 0, and ends at “infinity.” Now, we have enough information to solve the problem. Since temperature is a continuous function, there exists a time “c” such that , and f(c)=170, because 70<170<350. The temperature we seek is between the temperature the oven starts at, and room temperature. The time at which the oven is 170° is between time 0, and infinite time, therefore IVT would apply.
2. Let h(x) be defined by on the interval [-3,2]. Is there a place in this interval where h(x)=0?
   1. We have a closed interval, but in order for this to be an IVT problem, we must first check if the graph is continuous. Since it is a piecewise function, we must check if the function is continuous at the break points. There is a break point at x=0. Using substitution on the first piece of the piecewise function, we get h(x)=1. Using substitution on the second piece of the piecewise function, we get h(x)=-1. Since 1 and -1 do not meet, the function is not continuous, therefore IVT would not apply. There is not guaranteed to be a place on the defined interval in which h(x)=0, because the function is not continuous.

## Lola Tries

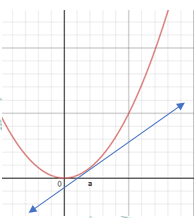
1. If on the interval [1,2] is there a place between x=1 and x=2 where f(x)=0?
2. Does take on the value 0.4999 for some t in [0,1]?
3. Does a root exist on the interval [0, ] for the function ?

Next Stop: Kenya

# Image result for kenyaImage result for nairobi national parkWelcome to Kenya!

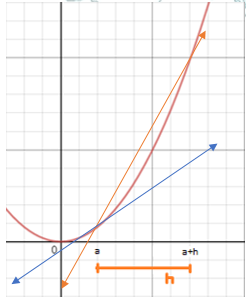
Karibu Kenya!

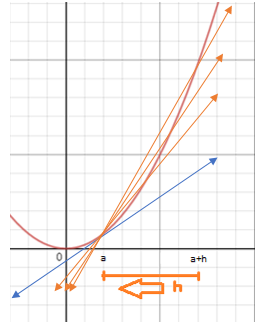
What is a derivative and the limit definition of a derivative?

Imagine a linear function, such as . What is the slope of this function? Well, we can determine that in several ways, whether it is the slope equation (, from the graph of the line, or the “m” value, as this function is in y-intercept form.

Now consider a nonlinear function, , shown to the right- what is its slope?

Since it is not a straight line, there is a different slope at every point along the function. How then would we find the slope at a specific point? One approach is to draw a straight line tangent to a given point, say, x = a. because we already know how to determine the slope of a straight line.

Now all we have to do is determine the slope of this tangent line. It would be . But wait a second- that would yield ! Therefore, in order to use this slope equation, we need more points.  
Although it won’t be exact, we can approximate the slope at a given point using a secant line, which is drawn between two points, say at our original point, at , and a point units away horizontally, at . This would mean that our slope becomes .

However, we want to be as accurate as possible when determining the slope at a given point. Thus, we must move our secant line a little closer to the tangent line, by reducing the value of , and effectively bringing the point closer to a.

Technically, we could bring closer to for eternity! But that would take a little too long on the AP Exam.Instead, we can represent this mathematically as , because we reduce further and further to bring closer to . If we make smaller and smaller, approaching 0, this also means that our secant line approaches the tangent line. Thus, determining is the same thing as determining the slope for the tangent line at a given point- or as we’ll call it from now on, **the derivative!**

Now we can formally define that the limit definition of a derivative allows us to the instantaneous rate of change along a curve- i.e., the slope at a given point. Furthermore, we can notate this in two different ways; Newtonian notation and Leibnitz notation.

**Newtonian notation**:   
**Leibnitz notation:**

Let’s take a look at an example. Given a nonlinear curve like , what is the instantaneous slope at x = 1? I.e., what is ? Now it's easy to figure this out using the limit definition of a derivative! Based upon this definition, we can solve for .

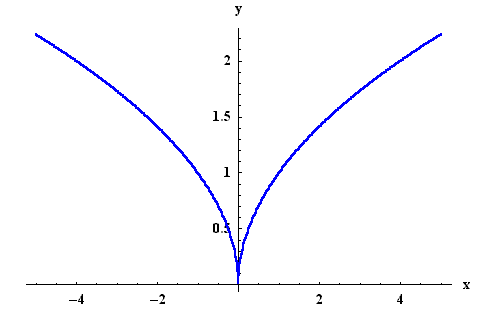
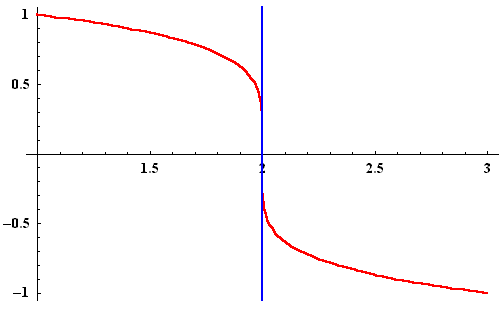
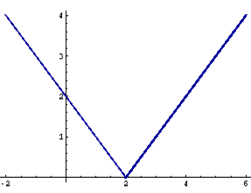
So, to recap- *we just figured out the slope at a* ***specific*** *point on a* ***nonlinear curve****!* But we can go even farther. Imagine that you wanted to find the slope at multiple points, say, at on a more complex curve, like . With our new limit definition of a derivative, this might seem like a piece of cake at first- but it soon becomes difficult to quickly solve for all of these points! Instead, there is a better way- we can define a **derivative function**, which will tell us the derivative at any x value along the curve. How can we do this? Simply plug in “x”, representing any x-coordinate on our curve, into the limit definition of a derivative.

Thus, , and this will gives us the slope of at any point! This saves us tons of time- let’s try plugging in those points from earlier!

Clearly finding a derivate function makes more complex problem solving much easier.

Let’s try another example problem. Take a function , and we want to find . If we plug this into the limit definition of the derivative, we would find the following:

Woah! For some reason, it appears as if we cannot determine the derivative at for - you could even say that this function is **non-differentiable** here. It makes sense that this function is non-differentiable at if we look at a graph; there is a vertical asymptote at this point, meaning that we cannot even determine the value of the function at this point, let alone the instantaneous slope.

This points to an important condition for differentiability; that the function must be continuous at a given point. However, there are two other states in which a function will not be differentiable. The first is when a corner or cusp occurs, shown on the right, where the slope of the function changes instantly. This means that there is a technically infinite number of possible slopes at that corner or cusp, making it impossible for us to determine a single slope at this point. The second time a function will be non-differentiable is when there is a vertical tangent, as shown on the left. This is because the derivative function will tend towards infinity, yet can never reach infinity, causing a vertical asymptote in the derivative function- making the derivative function discontinuous at the point of vertical tangency on the original function, and thus making the original function non-differentiable.

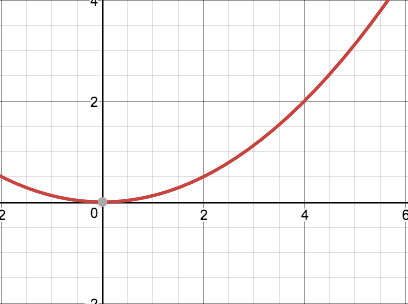
Corner

Cusp

## Guided Practice

1. Given , determine .
   1. asks for us to find the derivative, or the instantaneous slope, at x = 2 on . To find this, we can use the limit definition of a derivative. Therefore,   
      So, we found that the instantaneous slope at x = 2 is 0.
2. Given , what is ?   
   Remember that means the same thing as - thus, when this questions asks “what is ”, it is really asking for a function which gives the instantaneous slope at any value of x on . Therefore, we can solve this as follows:

## Lola Tries

1. From the graph of shown to the right, approximate by drawing a tangent line.
2. Given , what is ?
3. Given , what is ?

Next Stop: Nigeria

# Image result for nigeria tourist attractionsImage result for nigeria landmarksWelcome to Nigeria!

Barka da zuwa Nigeria!

What is the power rule, product rule and chain rule?

There are shortcuts to finding derivatives! If we had to use the limit definition of a derivative every time we wanted to find a derivative, not only would it be extremely inefficient, it would also make us hate calculus! Thus, there are three major shortcuts we can use to find a derivative!

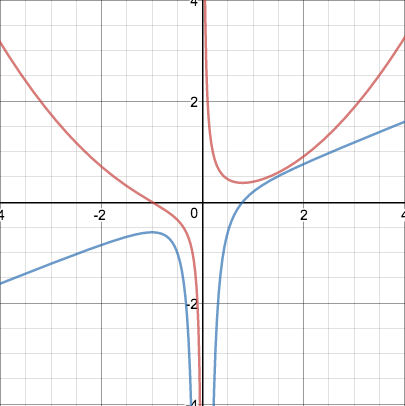
**Power Rule:** If n is a real number, then . We simply take the power on x and multiply it by the coefficient in front of x, and subtract 1 from the existing power.

If we had the function, , we would repeat this process for all three terms of our function (, and ) Let’s start with the first term of our function, . By power rule, we multiply 2 by the power on x, which is 2. The new coefficient would then become 4. Then, we have to subtract 1 from the power that originally existed on x, which is 2. 2-1=1. Therefore the term becomes when we derive. Then, we move on to the second term of the function, . We would multiply 8 by the power on x, which is 1. Since any number times 1 is itself, the coefficient remains 8. Then we would subtract 1 from the existing power. Since the power on x was originally 1, after subtracting 1, it becomes zero. Anything raised to the power of 0 is disregarded. Therefore, the term , simply becomes 8 when we derive. Lastly, we have the term 709,021. There is no x in this term! It is just a constant. When we derive a constant, it always goes to 0. This is because the power on x is originally 0, and so we would multiply this to the existing coefficient, 709,021. Any number times 0 is 0, so the constant just gets wiped away. Therefore, the term 709,021, simply becomes 0 when we derive. Now we just piece together the individual terms that we derived to get our overall derivative. Therefore, .

**Product Rule:** If f and g are both differentiable functions, then . When a function has two terms multiplied together, we take the first term and multiply it by the derivative of the second, and add that to the second term multiplied by the derivative of the first.

For example if we had the function , we first identify the two terms as f and g. Let’s say that and . Since both terms are continuous throughout their domain, and do not have any corners and cusps, they are differentiable. Thus, we can use product rule! First we would multiply f by the derivative of g. The derivative of x+3 is 1, by power rule. We multiply this by f left alone. . Then we multiply g by the derivative of f. The derivative of is , by power rule. We multiply this by g left alone. . Now we add the two parts together. Thus, we would have = .

**Quotient Rule:** If f and g are both differentiable functions, then . When a function has one term divided by the other, we take the derivative of the numerator, leaving the denominator alone and subtract that from the derivative of the denominator, leaving the numerator alone. All of this will be over the denominator squared.

For example, if we had the function , we first identify f and g. F is always the numerator, . G is always the denominator, . Since both terms are continuous throughout their domain, and do not have any corners and cusps, they are differentiable. Thus, we can use quotient rule! First we take the derivative of f and multiply it by g. The derivative of by power rule. We then have Then we take the derivative of g and multiply it by f. The derivative of 5x=5, by power rule. We then have . Now, we just square g, our denominator. . We take all of the individual components that we found and put them into . Therefore, we would have . Now all we have to do is simplify! . To the right, you can see the graph of the original function shown in red, and the derivative function shown in blue.

## Guided Practice

1. If , what is
   1. First, let’s identify which rule to use. Since two terms are being multiplied, we would use product rule. Now we must identify what f and g are. and . Since both terms are continuous throughout their domain, and do not have any corners and cusps, they are differentiable. Thus, we can use product rule!
   2. We first take the derivative of g. can be rewritten as We can use power rule! By power rule, we multiply ½ by the coefficient in front of x, which is 1. This becomes . Now, we must subtract 1 from the existing power to find our final derivative. This would give us . We then multiply this by our first term left alone.
   3. Now we take the derivative of f. Again, we use power rule! has three terms. We use power rule on each of these individual terms. The derivative of , because we take the power on x, 3, and multiply it by the coefficient that was already on x, 1. Then we subtract 1 from the existing power, 3-1=2. We then take the derivative of the second term, By power rule, the derivative of , because we take the power already on x, which is 2, and multiply it by the coefficient in front of x, which is 4. . Then we subtract 1 from the existing power, 2-1=1. Next, we take the derivative of the third term, 10x. The derivative of -10x=-10. Using power rule, we take the power on x, 1, and multiply it by the coefficient in front of x, -10. Then we subtract 1 from the existing power, 1-1=0. This wipes away the x. Now we piece together each of the terms to get the overall derivative of f. . Lastly, we multiply this by g left alone.
   4. Lastly, we add the two parts together.
2. If , what is
   1. First, let’s identify which rule to use. Since two terms are being divided, we would use quotient rule. Now we must identify what f and g are. F is always the numerator, thus and g is always the denominator, thus . Since both terms are continuous throughout their domain, and do not have any corners and cusps, they are differentiable. Thus, we can use quotient rule!
   2. First, we must take the derivative of the numerator, f. The derivative of can be found using power rule. With power rule, the derivative of any constant, in this case 3, goes to 0. The derivative of -2x=-2 because we would multiply the existing power on x, 1, by the coefficient, -2, and then subtract 1 from our power, 1-1=0. This would wipe x away. Thus, the derivative of f is simply -2. We multiply this by the denominator left alone. (-2)(4x+1)=-8x-2.
      1. Then, we must take the derivative of the denominator, g. The derivative of the denominator can also be found using power rule. With power rule, the derivative of 4x=4, because we would multiply the existing power on x, 1, by the coefficient, 4. The derivative of any constant, in this case 1, goes to 0. Thus the derivative of g is simply 4. We would multiply this by our numerator left alone. 4(3-2x)=12-8x
   3. Now, we must square the denominator, giving us .
   4. We take all of the individual components that we found and put them into . This would give us
   5. We have to take it one step further, as we want the derivative at x=8. We would simply plug 8 into the derivative function to get that. .

## Lola Tries

Next Step: Morocco

# Image result for moroccoImage result for moroccoWelcome to Morocco!

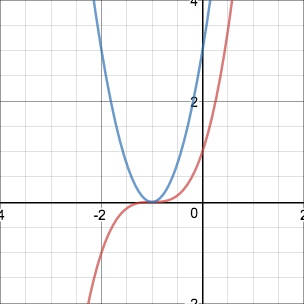
!مرحبا بك في المغرب   
What is the chain rule?

The power rule, product rule, and quotient rule are incredibly useful! However, what would we do if we wanted to derive a composition of a function, f[g(x)]? In this instance, one function’s output becomes the other’s input. This is often referred to as a “function in a function.” We can use chain rule to derive composite functions!

The chain rule states, if f and g are both different functions and F= f ○ g is the composite function defined by, then F is differentiable and F’ is defined by the product:

What on earth does that even mean? Basically, this rule states that you do the derivative of the outside piece, leaving the inside piece alone, and then multiply it by the derivative of the inside piece.

How would we find the derivative for a function like ? The first idea that probably comes to mind is to foil out , and solve it using the power rule from there. However, how would we find the derivative of a function like ? Although, technically, we could foil this out, it would literally take hours. So, chain rule is extremely effective here!

Let’s revisit the example; . First, let’s identify the outside piece and the inside piece. The inside piece would be , as this is the function inside of the cube. Thus, the outside piece would be , and our entire function would be . With chain rule, we take the derivative of the outside piece first. . However, since g(x) is the input for f(x), we would replace the x in with g(x). Thus, . Now, we multiply by the derivative of the inside piece, g(x). The derivative for is just 1, by power rule. . Therefore, .

Think of the chain rule as peeling an onion. Like an onion, we use chain rule with composite functions that have multiple layers. First, you take the derivative of the outer layer. Then you have to multiply this by the derivative of the inner layer. On the right, the graph of the original function is shown in red, while the graph of the derivative is in blue.

## Guided Practice

1. Find the derivative of
   1. First, let’s identify the rules we need to use! Since the overall function is a fraction with differentiable functions as the numerator and denominator, we can use quotient rule. However, we must also use chain rule because the numerator is a composite function itself! The outside piece would be and the inside piece would be , with . The same is true for the denominator. It is a composite function as well! The outside piece for the denominator would be and the inside piece would be , with .
   2. Now that we have the fundamental ideas down pat, we can start to derive. With quotient rule, we first take the derivative of the numerator and multiply it by the denominator left alone. The derivative of the numerator would use some chain rule. The derivative of the outside piece would be . Then we multiply this by the derivative of the inside piece. The derivative of by power rule. Thus, the derivative of the numerator would be . Lastly, we multiply this by the denominator left alone, giving us
   3. Now, we must subtract the derivative of the denominator times the numerator left alone. The derivative of the denominator would also use some chain rule. The derivative of the outside piece would be , the derivative of the inside piece would be just 1, by power rule. Thus, the derivative of the denominator is . Lastly we multiply this by the numerator left alone, giving us .
   4. Now, we have the numerator of the overall derivative. All we have to do is simplify. =  
       =
   5. Now all we have to do is square the denominator, to get the denominator of the derivative! This would give us
   6. Now we just have to piece together the entire derivative!
2. ; What is ?
   1. First, we need to identify which rules to use. Since there are two functions multiplied together, and they are both differentiable, we can use product rule! However, we must also use chain rule because the first term in the function is inside of another function, making it a composite! The outside piece would be and the inside piece would be , with . The same is true for the second term. It is a composite function as well! The outside piece for the denominator would be and the inside piece would be , with .
   2. Now we can start deriving. By product rule, we take the derivative of the second term and multiply it by the first term left alone. The derivative of the outside piece of the second term would be . We then multiply this by the derivative of the inside piece. The derivative of , by power rule, so we are left with . Finally, we multiply this by the first term left alone,
   3. Next, we take the derivative of the first term and multiply it by the second term left alone. The derivative of the outside piece of the first term would be . We then multiply this by the derivative of the inside piece. The derivative of , by power rule, so we are left with . Finally, we multiply this by the first term left alone, .
   4. Lastly, we must piece together the overall derivative and simplify.

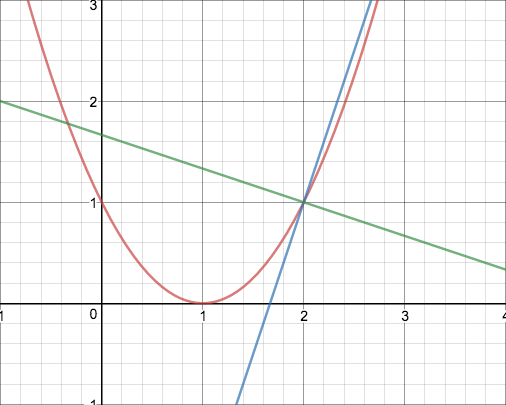
Next Stop: Egypt

# Image result for egyptImage result for egyptWelcome to Egypt!

مرحبا بكم فى مصر!

How do I find the equation of a tangent and normal line to a curve at a point?

All this time we have discussed how to find the slope of a line tangent to a curve at a specific point. However, what if we wanted to determine the actual equation for that tangent line, not just its slope? To do this, we will need our slope intercept form, . If we start out with a curve such as , and want to determine the tangent line at x = 2, we can already plug in , as our x-value is 2. As well, another easy value to find , as all we need to do is plug 2 into , which in this case would be 1. Finally, we need to determine , the instantaneous slope at this point; a.k.a., the derivative! Using the chain rule we just learned, is , which simplifies to become . At the point , . Therefore, the equation for the tangent line at is . Most of the time this is sufficient, however, we can rewrite it as to graph it.

Another type of line that we can determine in a similar fashion is a “normal line”. A normal line is one which is perpendicular to the tangent line as graphed on the right.

To determine a normal line, we first follow the same exact steps taken to determine the tangent line. However, when we determine the value of , we must take the derivative at our specific point, and then take the negative reciprocal of this slope (i.e., the slope of the normal line = ). Let’s look at this with our previous example. If we wanted to find the normal line at the point , we would use the x and y coordinates determined earlier on (2 and 1 respectively), and then take the negative reciprocal of , which would be . Thus, the equation for our normal line would be .

## Guided Practice

1. Find the line tangent to at .
   1. To determine the tangent line, we need the point of tangency and the instantaneous slope at that point. We already have the x-value of this point, and thus we can determine that the y-value is . Next, we can determine the instantaneous slope by finding the derivative function and plugging the x-value into this function. The derivative function, per power rule, would be . When we plug in , we find that the instantaneous slope is . Therefore, our tangent line would be .
2. Find the normal line at on
   1. The first step to find a normal line is the same as finding a tangent line; we must determine both the x and y coordinate of the point on that the normal line intersects. In this case, the y-coordinate at would be . The second step to find a normal line is to find the negative reciprocal of the instantaneous slope; i.e., . The derivative function in this example would be , and thus the instantaneous slope at is . Then, the negative reciprocal, and the slope of the normal line, would be . Therefore, the equation of the normal line at on would be .

## Lola Tries

1. Find the line tangent to the curve at .
2. Find the line tangent to the curve at .
3. Find the normal line on the curve at .

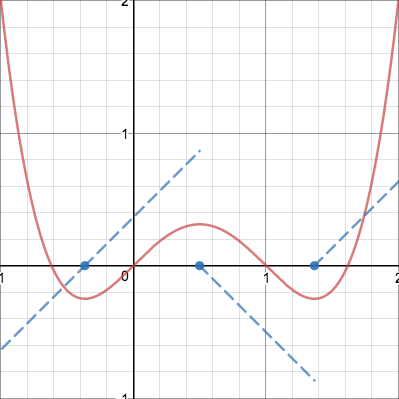
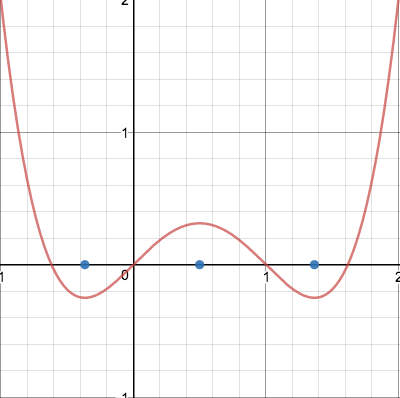
Next Stop: Ethiopia

# Image result for ethiopian cultureImage result for king lalibela churchWelcome to Ethiopia

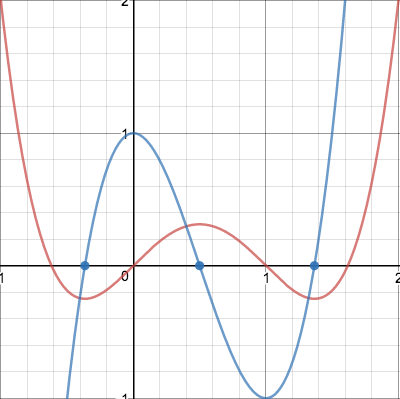
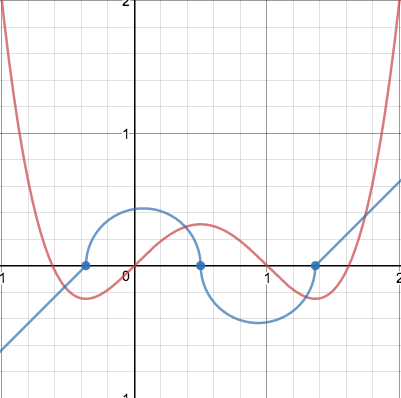
ኢትዮጵያ ወደ አቀባበል!

How do I graph the derivative from a drawn graph and vice versa?

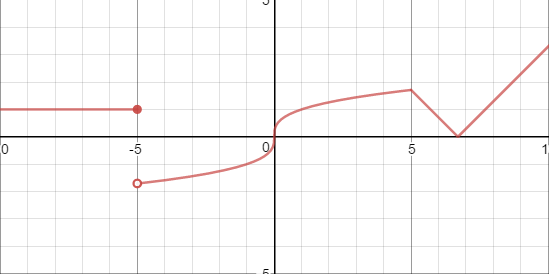
Let’s say that we are given the graph of a function , and want to find the graph of the derivative of this function, . However, we are not given the actual function itself, only its graph.

To graph the derivative function, there are three main steps we must go through. First, it is important to identify where . Generally, this will be when the graph of reaches some sort of maximum/minimum or simply flattens out at a point, as the slope at these points will be 0 and the tangent line will be perfectly horizontal.

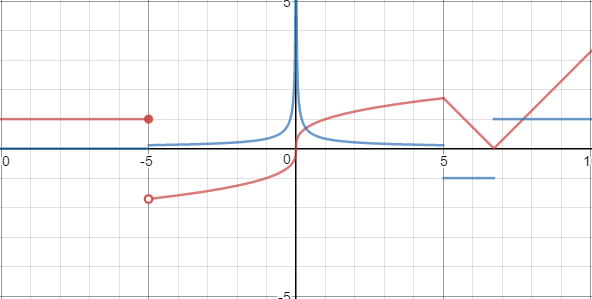
Second, it is important to identify the areas where and where . Since is a graph of the instantaneous slope at a given x, this means that we can divide the graph into domains where is increasing and where is decreasing. When is increasing, , and when is decreasing, .

After we’ve identified these regions, we can then begin to draw a general sketch, drawing a graph following the parameters determined in steps 1 and 2.

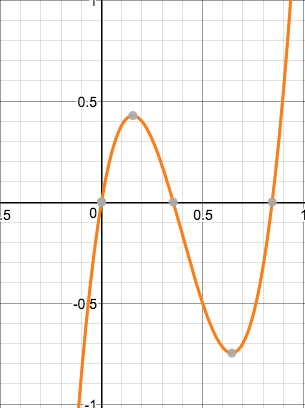
Third, it is important to identify how steep the slope of generally is at a given point. For example, when has a very steep upward slope, the y-value of will be relatively much higher. As well, when is locally increasing with the steepest slope, will have a local maximum. Likewise, when is locally decreasing with the steepest slope, will have a local minimum.

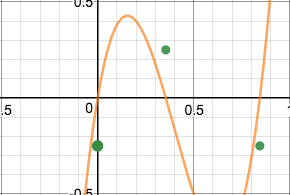
This works for most functions, however, when determining the points where in step 1, it is also important to consider the places where is non-differentiable. Let’s look at the graph to the right, which has all three conditions for differentiability.

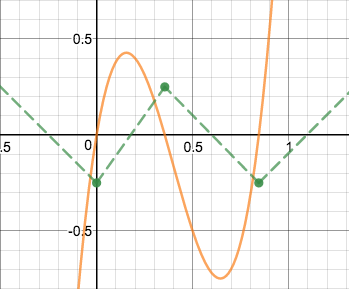
Whenever there is a discontinuity, corner, or cusp, such as at , or the corner of the absolute value function on the right, the derivative graph will usually have a jump discontinuity. This is because these trends on the original graph represent instantaneous, and thus discontinuous, changes in the slope. When there is a vertical tangent, such as at , the derivative function will have an infinite discontinuity, as from either side of the vertical tangent the slope of the function approaches infinity. In this example, the derivative function will approach infinity from either side of the vertical tangent because from either side the function is continually going “up.” If it were flipped about the x-axis, and the function was continually going down, then the derivative function would approach negative infinity from both sides. Understanding these rules, we can go ahead and graph the derivative of this function on the right.

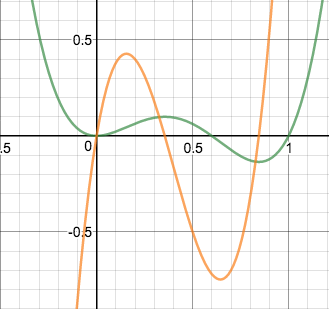
The derivative function is graph in blue, and it is rife with the discontinuities we predicted above. At , it may be hard to see, but there is a discontinuity, which is the result of the jump discontinuity between the line of and . Then, there is an infinite discontinuity, at , as the result of the vertical tangent there on the original function. Finally, we see several jump discontinuities at and , due to the presence of corners and cusps and these locations.

Given these skills and information, it is also possible to go “backwards,” and graph the original function from the graph of the derivative! Let’s look at this using our first example.

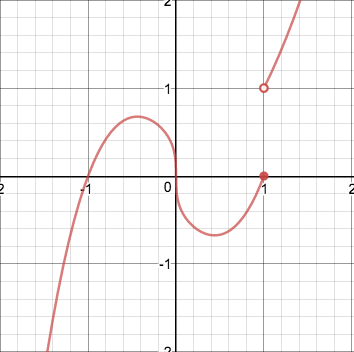
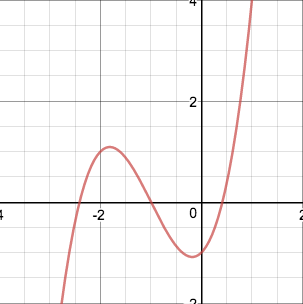
 Given that the graph to the right is the graph of the derivative function, we can start off by determining the maximums/minimums of the original graph. Since the x-intercepts of the derivative function occur whenever the original function has a slope of zero, we can say that these points are likely the maximums/minimums of the original function. We can confirm this by the behavior of the derivative function on either side of these points. If the derivative function goes from positive to negative, then it is likely a maximum on the original graph, as a maximum will occur at the point where the function increases, stopped changing instantaneously at the maximum, and then decreases. Likewise, a minimum point on the original function will occur when the derivative function goes from negative to positive, as this means the original function decreases, stopped changing instantaneously at the minimum, and then increases.

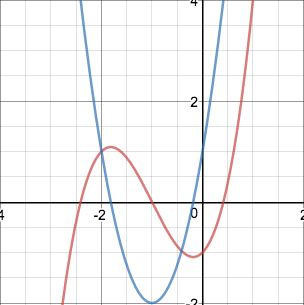
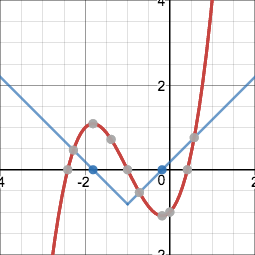
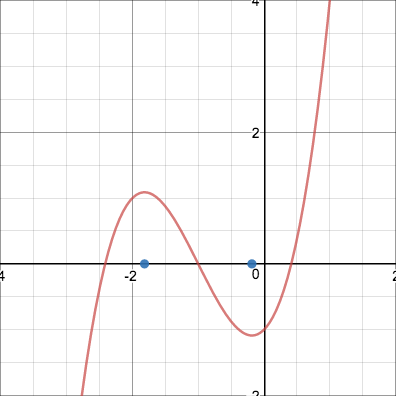
 Therefore, we can sketch out these maximum/minimum points generally, even if we do not know the exact value they are at on the original function.

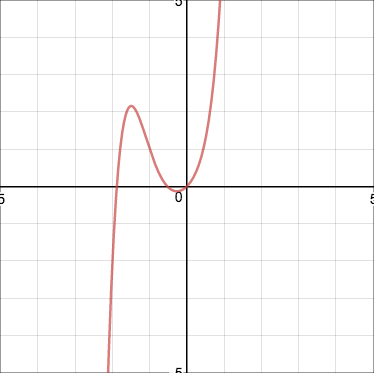
 Next, we can sketch out the general pattern of the original function. For example, the derivative function describes that the original function, on , had a negative slope, as the derivative function is entirely below the x-axis on this interval. Therefore, we know that as the original function was approaching the minimum at , it was going downwards. Between the minimum at and maximum , the derivative function describes a positive slope, and so the original function must have generally been going up between these points. From the maximum at and the minimum at , the derivative function was again negative, and so there is a downward slope between these points. Finally, after the minimum at , the derivative function is positive on , meaning that the original function continually rises past this point.

 Finally, after we have sketched this general trend, there is one more factor that we must take into account- the actual steepness of the original graph at various points. When the derivative function reaches a maximum, this means that the slope of the original graph was the most “positively steep”- i.e., the original function was increasing the fastest. Similarly, when the derivative function reaches a minimum, this means that the slope of the original graph was the most “negative slope” and decreasing the fastest. With this information, we can more accurately sketch the original function. Note, the following original function graph, in green, was done using a calculator- your own graph should be accurate in terms of the general trends, but it does not have to mirror the actual points of the original function!

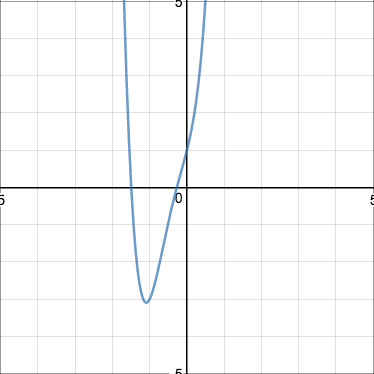
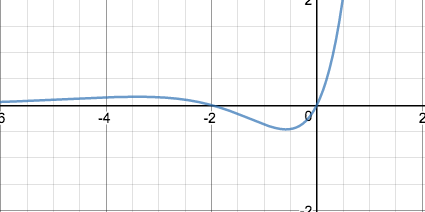
## Guided Practice

1. Given the following graph, identify any possible discontinuities in the derivative function.
   1. Based upon this graph, there are two discontinuities. The first is at . This is because at there is a vertical tangent line, and thus, the derivative function would have an infinite discontinuity, where the derivative function approaches negative infinity from both sides (as the function has a negative slope on either side of ). As well, there is a second discontinuity in the derivative graph, at , as there is a jump discontinuity in the original function. Therefore, the slope of the original function changes instantaneously, making the derivative graph discontinuous.
2. Given the following graph, graph the derivative.
   1. First, we must make sure to graph the x-intercepts of the derivative function. Next, we sketch a general shape to the graph, per the intervals on which it is increasing/decreasing. Finally, we can refine our graph by considering how steep the original function is at different points, to successfully graph the first derivative.





## Lola Tries

1. Given the red graph, graph the derivative function.
2. Given the blue graph to the left below, graph the derivative function.
3. Given the blue graph of the derivative to the right below, graph the original.

Next Stop: United Arab Emirates

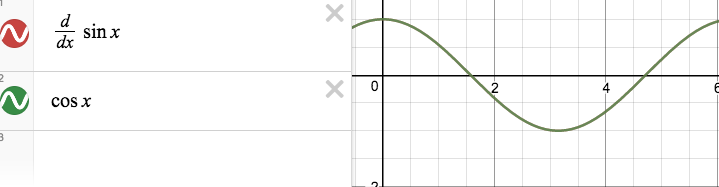
# Image result for united arab emirates desertImage result for burj khalifaWelcome to the United Arab Emirates!

متحدہ عرب امارات میں خوش **آمدید!**

What are the derivatives for trigonometric functions?

Let’s get triggy! We can take derivatives of trigonometric functions! In fact, there is a definitive rule for the derivative of each trigonometric function!

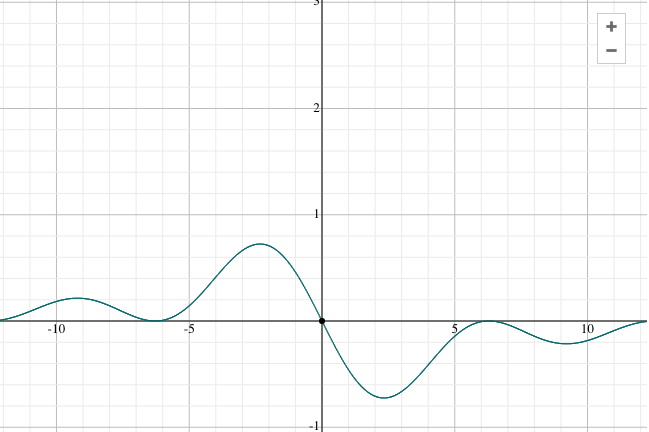
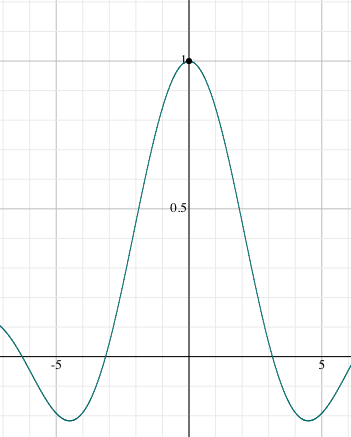
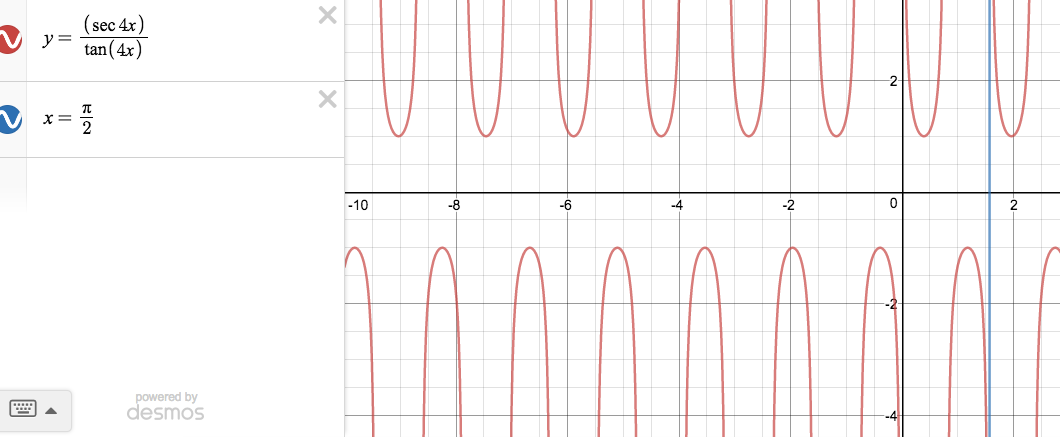
The derivatives of these trigonometric functions can be used in many useful applications. But, where did they come from? All of the trigonometric functions can be proven through some algebraic manipulation. Look at the guided practice problems for one of these proofs!

For a visual, the derivative of y=sinx is cosx. The graph shows this in action!

These trigonometric derivatives are very useful for real-life applications. For example, Lola went sky diving off of the Burj Khalifa. The angle of elevation from her parents is modeled by the function , in which x is the time in seconds, and y is the angle of elevation in radians. What is the rate of change in the angle of elevation at seconds?

First we must derive the function to get the rate of change in the angle of elevation. Using chain rule, we have to take the derivative of the outside piece, leaving the inside alone. The derivative of is . We are left with . We then multiply this by the derivative of the inside piece. The derivative of is . So, . We can then plug in seconds. .

## Guided Practice

1. Prove that
   1. First, we must put this into the limit definition of a derivative. By doing so, we yield .
   2. Then, we must recall that . Thus, we can separate cos(x+h) using this rule! We are left with .
   3. Next, we can separate this limit into two different limits. Since the limit is as h approaches 0, we can evaluate the limit as . To evaluate both of these limits, let’s look at the behavior of the graph as h approaches 0.
   4. , as you can see in the first graph to the right, because as h approaches 0, the y-value is getting closer and closer to 0.
   5. , as you can see in the second graph to the right, as h approaches 0, the y-value is getting closer and closer to 1.
   6. Thus, we are left with , which leaves us with
2. Find the equation of the tangent line for at x=.
   1. First, we must find the derivative of the function. To make it easier, let’s rewrite it with sines and cosines. .
   2. Now, we derive! We can easily use our rule for the derivative of Since there is a 4 inside of the cscx, we must multiply by four, to comply by chain rule. So we are left with,
   3. Next, we can plug in into the derivative function to get the slope at that x-coordinate. Thus, there is no slope that is tangent to that point. There is no line tangent to at x=. Got you!!
   4. As you can see from the graph, there is no defined slope of the line tangent to x=, as there is a vertical asymptote there.

## Lola Tries

1. Prove that
2. Find given
3. Find given

Next Stop: Saudi Arabia

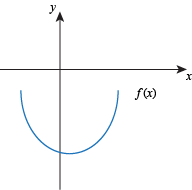
# http://static.panoramio.com/photos/large/9225136.jpghttps://ccmtheimaginarytraveler.files.wordpress.com/2014/09/oryxsa-files-wordpress-com.jpgWelcome to Saudi Arabia!

مرحبا بكم في المملكة العربية السعودية!

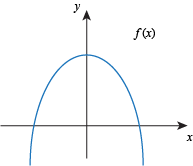
What is the second derivative? What are intervals of concavity and points of inflection?

Now that we understand the derivative, we can explore **higher order derivatives!** Take a function, . Then, find its derivative- i.e., . Remember that even though is the derivative of , it is still a function itself, and we can determine the instantaneous slope on it as well. To do this, we would have to take the derivative of the derivative, also known as the **second derivative!** Therefore, , the second derivative, would be 2, per power rule. And again, is also a function which we can take the derivative of- the third derivative.

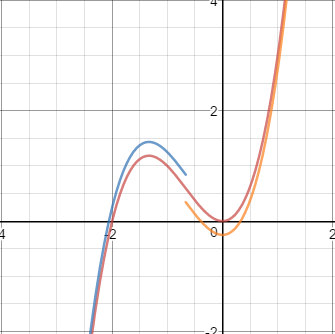
Before we continue, it is important to know the basic notation for higher order derivatives.

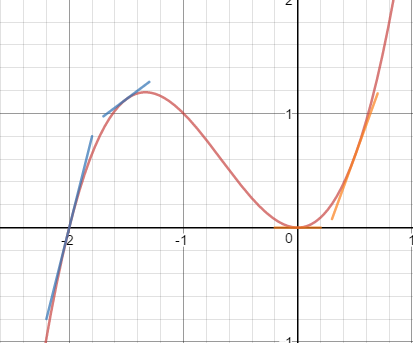
**Newtonian:** For the first, second, and third derivatives, there is n-number of apostrophes, whereas any derivatives greater than third are notated as , where n is the order of the derivative. For example, the 4th derivative is   
**Leibniz:** Any derivative greater than the first is notated as , where n is the order of the derivative. For example, the 4th derivative is .

For now, we are going to focus on the importance of the second derivative. The second derivative describes a property of the original function known as “concavity.” When a function is “concave up like a cup” any tangent line to the curve will be below the curve, , and the value of is increasing (as describes the slope of ).

When a function is “concave down like a frown”, any tangent line to the curve will be above the curve, and , and is decreasing.

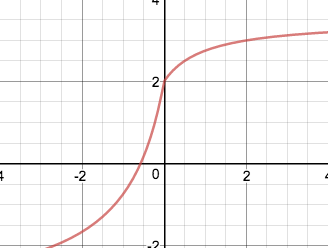
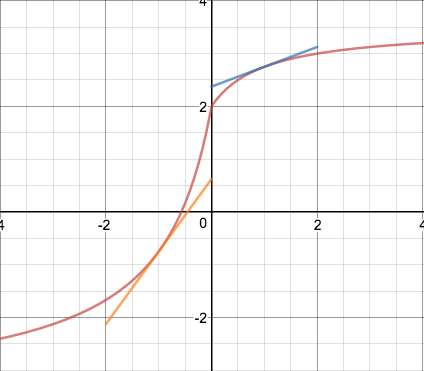
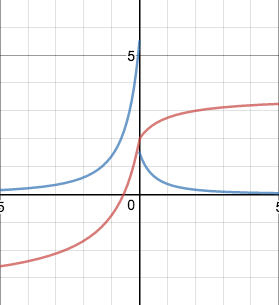
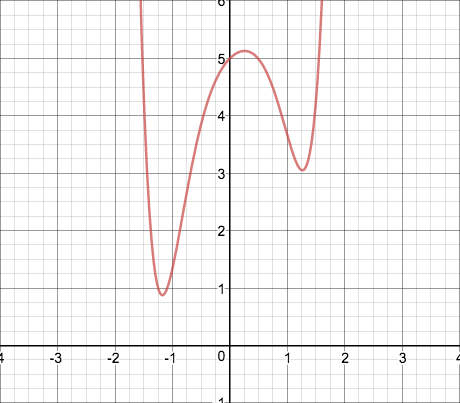
As well, a “point of concavity” is simply when the concavity of a function changes. Since is the slope of , these points of concavity occur when is at a maximum or non-differentiable and changes from increasing to decreasing or vice-versa, or when is 0 or discontinuous, as this represents a continuous/instant change in the sign of , and thus the concavity.

Generally, it is possible to approximate these intervals of concavity by understanding the basic ideas of concavity.

For example, given the graph to the left, we can highlight when it is concave up/concave down. From the left end of the function (it’s end behavior on the left) until about , the function is concave down because it is “like a frown”, and the tangent lines are above the curve, while past , the function is concave up, as the shape of the graph is “like a cup” and the tangent lines are below the curve, as seen in the following graph.

Therefore, we can generally say that there is a point of inflection around , and that from the function is concave down, while from , the function is concave up. Note that we do not include , as ) is zero, as it is our estimated point of inflection.

## Guided Practice

1. Given the function graphed to the right, determine the intervals of concavity and the point(s) of inflection.
   1. Based upon the graph of this function, it appears as if on the left side of the graph, it is concave up, as it is “up like a cup.” On the right side of the graph, it appears as it is concave down, as it is “down like a frown.” As well, when the graph is zoomed in, it appears as if there is a cusp, acting as both a sudden change in the slope and concavity of the function at , which indicates that this is the point of inflection for the graph. Therefore, it is possible to say that the function is concave up o n and concave down on , with a point of inflection at .
2. Using the function in the previous problem and the intervals of concavity, draw a sketch of the derivative function.
   1. An extra application of concavity is that it allows us to draw more accurate derivative functions! This is because concavity describes the slope of the derivative function. Therefore, we can say that on , the slope of the derivative function is positive. More importantly, the graph of the original function shows that it becomes more and more concave up as the function approaches from the left, and then the concavity abruptly changes, meaning that the slope of the derivative function grows greater and greater approaching this point, before it abruptly changes as well. Similarly, as the function approaches from the right, the function becomes more and more concave downward, indicating that the slope decreases faster in this area. Using this information, we can sketch a more accurate derivative function, which should look close to the graph on the right.

## Lola Tries

1. Given , determine and .
2. What is the definition of “concave up” and “concave down”?
3. Approximate the intervals of concavity and point(s) of concavity on the graph to the right.

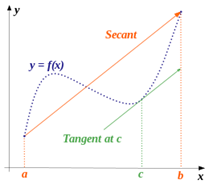
Next Stop: Turkey

# Image result for pictures of turkey countryRelated imageWelcome to Turkey!

Türkiye'ye hoşgeldiniz!

What is the Mean Value Theorem and Rolle’s Theorem?

The Mean Value Theorem is basically the intermediate value theorem, but for slope!

**Mean value theorem:** If f(x) is continuous on the interval [a,b] and differentiable on (a,b), then there is at least one value “c” on the interval (a,b) that is a<c<b, such that .

In other words, the slope of the line through (a, f(a)) and (b, f(b)) is the same as the tangent line of some point “c” on the interval (a,b). Look at the graph for a visual description!

There are a few steps with Mean Value Theorem problems:

1. Check for continuity
2. Check for differentiability
3. Find slope of secant line
4. Determine where f’(x)=slope of secant line
5. Determine which x’s are in given interval.

For example, determine if the hypotheses for Mean Value Theorem is satisfied for on the interval [1,3]. If so, find c. If not, tell why.

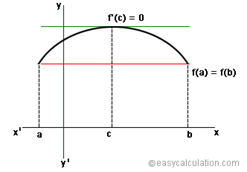
Since f(x) is a simple polynomial function, it is continuous on the interval! Then, to check differentiability, we must first find the derivative function. Using power rule, . Again, since this is a simple polynomial function, we know it is differentiable on the entire interval!

Now, we must find the slope of the secant line. , and while , making .

Now, we set this slope equal to to f’(x) to find where f’(x)= slope of secant line. ,

Since x=2 is on the interval from [1,3], we know at c=2, the instantaneous slope The graph shows this in action! The original function is in red, the secant line is in orange, and the tangent line at x=2 is in blue.

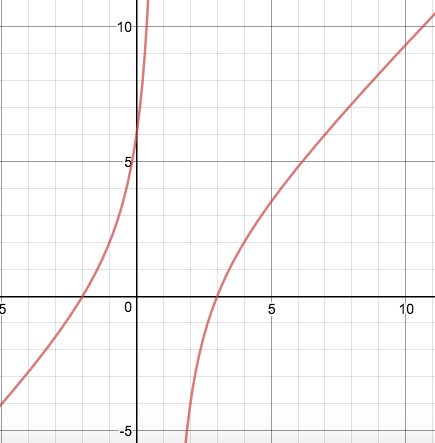
Rolle’s Theorem is simplify a specific application of Mean Value Theorem!

**Rolle’s Theorem:** If f(x) is continuous on [a,b] and differentiable on (a,b), and if f(a)=f(b), then there is some “c” in the interval (a,b) that f’(c)=0.

In other words, if f(x) is continuous and differentiable on an interval (a,b) and if f(a)=f(b), then at some point there will be a maximum or minimum or other point with a zero slope on that interval.

Like Mean Value Theorem, there are a few steps with Rolle’s Theorem problems:

1. Check for continuity
2. Check for differentiability
3. Check if f(a)=f(b)
4. Set f’(x)=0, to determine where slope is 0
5. Check for values of x where f’(x)=0, on the given interval.

For example, determine if the hypotheses for Rolle’s Theorem are satisfied for on the interval [-1,4] . If so, find c. If not, tell why.

First, we must check for continuity on the interval. is not continuous on the entirety [-1,4], specifically at x=1, because if we plug x=1 into f(x), we get

Thus, Rolle’s Theorem does not apply in this situation, as it is not continuous throughout the interval. Since this hypothesis is not met, we can stop there! On the graph, you can see the discontinuity at x=1.

## Guided Practice

1. Determine if the hypotheses for Mean Value Theorem is satisfied for on the interval [0,2]. If so, find c. If not, tell why.
   1. First, we must check for continuity. The only point of discontinuity is at x=-1, as this would yield a constant over 0. Since x=-1 is not on the interval, this is ok. All other points on the interval from [0,2] are continuous.
   2. Next, we must check for differentiability. To do so, we must first find the derivative using quotient rule. . We already know that the point at x=-1 is not differentiable, as it is not continuous. Again, since this is not on the interval from [0,2], this is ok. There are no other points of non-differentiability.
   3. Now, we must find the slope of the secant line, where . We can calculate that and , making .
   4. We then set this slope equal to f’(x) to find where f’(x)= slope of secant line.
      1. We can then plug this into the quadratic formula!
      2. For , we can only use the positive version, as the negative version is not in the given interval. Therefore,
2. Mean Value Theorem and Rolle’s Theorem also work with trigonometric functions! Determine if the hypotheses for Rolle’s Theorem are satisfied for on the interval [0, π]. If so, find c. If not, tell why.
   1. First, we must check for continuity on the interval. We know the nature of the function f(x)=sinx is continuous everywhere, so it would definitely be continuous from [0, π].
   2. Next, we must check for differentiability on the interval. The first step in doing so is to find the derivative function. Using our trigonometric function derivatives, we know that Since f’(x) is continuous everywhere, we know that f(x) is differentiable everywhere as well (including the interval from 0 to π)
   3. Now, we must check if f(0)=f(π). We find that and ; since f(π)=f(0) Rolle’s Theorem applies!
   4. Next, we must see where . Setting , we find . Lastly, we must check and see if these two x-coordinates are on the interval [0, π]. is not on this interval. So, , and at .

## Lola Tries

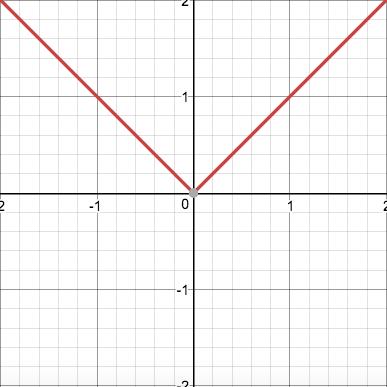
1. Determine if the hypotheses of Rolle’s Theorem are satisfied for on [0,1]. If so, find c. If not, tell why.
2. Determine if the hypotheses of Mean Value Theorem are satisfied for on   
   [-1,1]. If so, find c. If not, tell why.
3. Determine if the hypotheses of Mean Value Theorem are satisfied for on [2,5]. If so, find c. If not, tell why.

Next Stop: Iran

# Image result for iranImage result for iranWelcome to Iran!

به ایران خوش آمدید!

How do I find local maxima and minima using the 1st and 2nd derivative tests?

When graphing derivatives, we said that the local, or relative, maximum or minimum of a graph will generally be an x-intercept of the derivative graph. Essentially, this says that a function will have a slope of zero at a maximum or minimum; this makes sense when picturing the maximum of a parabola, as it levels off and has a horizontal tangent line right at the maximum.

Fundamentally, on a continuous and differentiable function, the derivative will be zero at local maxima and minima because there is a sign change in the derivative. For example, on the left side of a minimum point, the slope will be negative, and on the right side it will b e positive. Even if the graph is not differentiable at the local maximum or minimum, there is still some sort of sign change. The function is non-differentiable at , as there is a corner, yet is a local minima because there is an instantaneous sign change in the derivative; on the left of , the slope is -1, and on the right it is .

In order to put these principles into practice, we can use the **First Derivative Test**. This test involves the creation of a sign chart for the derivative of a function, in order to determine the sign of the derivative on either side of a “critical point,” the points where the derivative is either 0 or does not exist.

Let’s try this out on the function . First, it is necessary to find the critical points for this function. Without doing any calculations, it is possible to tell that there will be no critical points resulting from non-differentiability, as polynomials are continuous and differentiable for all real numbers. However, critical points can also occur where the derivative is 0. The derivative of would be , and setting this equal to 0 yields the following, , and thus .

Therefore, we have two critical points at and . Next, we must set up a sign chart for and ascertain the sign of on either side of the critical points. To the left of , we can plug in -2 (), which will yield a positive value. Between the critical points, we can plug in -1 (), which will yield a negative value. Finally, to the right of we can plug in 1, which will yield a positive value.

Based upon this sign chart, it is possible to tell that there is a local maximum at , as the slope goes from positive to negative, and a local minimum at , as the slope goes from negative to positive.

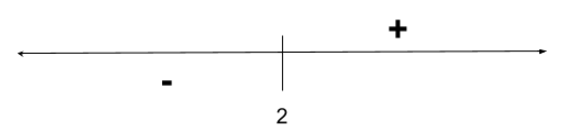
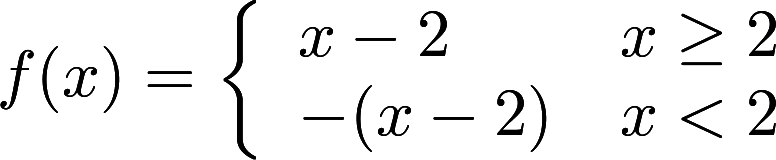
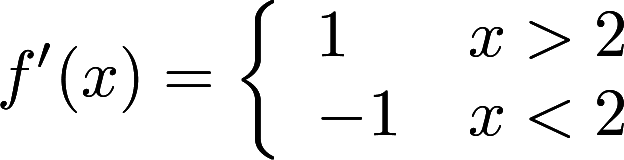
Yet the first derivative test is not the only way to determine local maxima and minima. We can say that when a curve is concave down and the slope is 0, there will be a local maximum, and when the curve is concave up and with a slope of zero, there will be a local minimum. This is based upon the principle that local maxima and minima occur when there is a sign change in the first derivative. For the sign of the first derivative to change requires that the slope goes from negative to positive, or positive to negative, this requires that the first derivative itself has a positive or negative slope, respectively. Thus, since the second derivative is the derivative of the first derivative and tells us the slope of the first derivative, if the function is concave down (), meaning that the slope of is less than 0, and , there is a maximum, while if the function is concave up (), meaning that the slope of is greater than 0, and , there is a minimum.

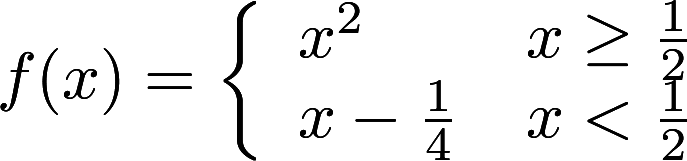
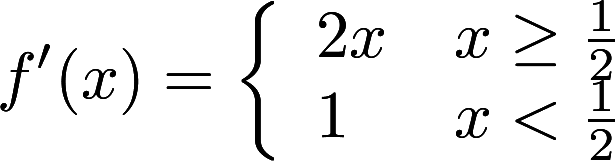
However, it is important to note that this approach does not work in two cases. The first is if does not exist at the given point, as this means that will not exist either. As well, it will not work if , as this does not tell us if the graph is concave up or down at the given point.

All of this information leads us to the aptly named **Second Derivative Test** which states that, given a function , there will exist a local maxima when and , given that is continuous and differentiable and that point, and a local minima will exist when and , given the same conditions. If, for some reason, is not differentiable at the given point (such as at a corner), or (at a point of inflection), we will have to use the first derivative test.

Let’s use the second derivative test with the same function that we used for the first derivative test, . First, it is important to determine where . As we already know, , and when or 0. Next, for the second derivative test, we need to plug these points into and determine the concavity at these points. In this case, . Plugging in yields -4, indicating that the concavity is downward at this point and that it is a local maximum. Plugging in yields 4, indicating that the concavity is upward at this point, and that it is a local minimum. Thus, the second derivative test is another effective tool we can use to determine local maxima and minima!

## Guided Practice

1. Given the function , identify any local maxima or minima.
   1. Whether using the first or second derivative test, it is important to find the critical points, where is either 0 or undefined. To find the derivative of an absolute value function, we will have to treat it as a piecewise function, as there is a corner where the slope instantaneously changes, and thus the derivative function is not continuous at that point. In practice, consider how a piecewise function, such as is essentially composed of two linear functions, meeting at . Right of , there is a line with a slope of +1 (), while left of there is a line with a slope of -1(. Therefore, can be represented by (piecewise garbage). Similarly, we can and represent it by , as there is a corner at (as this function is shifted two to the right horizontally), and left of this point is a line of (the same line as on the right, but with a negative slope) and on the right there is a line . Finally, we can derive this! Deriving each part of the piecewise separately, we would find that . Note that we do not include in either domain, as . Now, we must determine the critical points. There is one possible critical point at , as does not exist. As well, we can attempt to set both parts of the derivative function equal to 0 to find other critical points- however, there is no point at which the lines or equal 0; therefore, the only critical point is at . Finally, we can do the first or second derivative test to determine if this is a local maxima/minima. The second derivative test won’t be useful here, as is undefined, meaning is undefined as well.
   2. Therefore, since the sign of the first derivative changes from negative to positive at , there is a local minima at .

1. Given the piecewise function , determine the coordinates of any local maxima or minima.
   1. Since this is a piecewise function, just as we saw in the first example, we must find the derivative by taking the derivative of each part independently. Therefore, . Next, we must determine the critical points! We can first test for critical points when by setting each piece of the derivative equal to 0. For , , and for , when - however, this is not in the given domain of , so we throw this point out. As well, we can test for critical points when. On all of and , these functions are differentiable. However, there is a possible break in the overall piecewise function at . Therefore, we must test for continuity on both the original function and the derivative function, to determine if it is continuous and differentiable at this point. Following the three-step process to determine continuity outlined earlier, we find that our function is continuous and differentiable at every point- indicating that there is no point where . Thus, there are no critical points anywhere on this function, and no local maxima or minima.

## Lola Tries

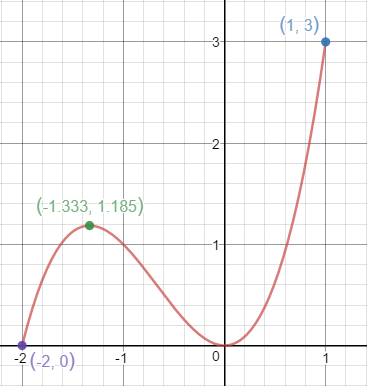
1. Determine the coordinates of the local maxima and minima for on [-5,3]
2. Determine the critical points for .
3. Determine x-coordinates of the local maxima and minima for .

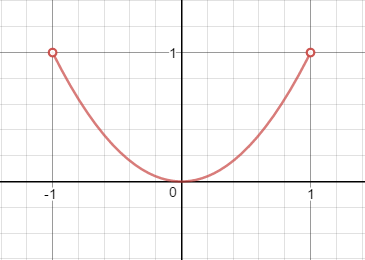
Next Stop: India

# http://cdn1.buuteeq.com/upload/18138/india-tours-jaipur-gangaur-festival.jpg.1340x0_default.jpghttp://www.luxurysociety.com/media/uploads/de/a9/dea9a4f8-07c6-4f57-96ba-b91752181a31/india-.jpgWelcome to India!

भारत में आपका स्वागत है!

What is the Extreme Value Theorem?



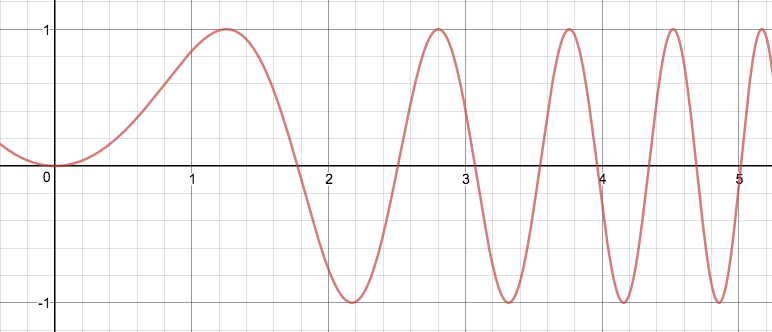
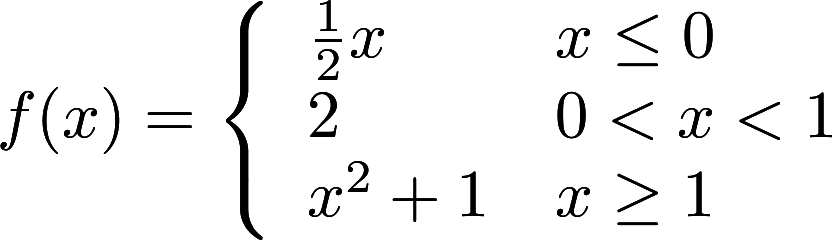
Up to this point, we’ve only discussed **local** maxima and minima; essentially, points which are the maxima or minima on very small, or “local”, intervals. However, these points are not necessarily the highest or lowest point on a graph, whether across the entire function, or even a specific interval. Local maxima and minima are not the same **absolute** maxima and minima- points which are *absolutely* the highest/lowest on an interval. For example, on the graph to the right, there may be a local maximum at (-1.333, 1.185), however, the absolute maximum is at (1, 3), and the absolute minima is at (-2, 0).  
 These absolute values are also known as extreme values, and they are governed by the **Extreme Value Theorem**. This theorem states that if a function, f, is continuous on the closed interval [a, b], then f must achieve an absolute maximum and an absolute minimum on [a, b]. These extreme values will either occur at the endpoints (a and b) or at a critical point on [a, b]. In less math-y terms, this basically means that if a function is continuous on a closer interval, there has to be an absolute maximum and minimum value. It is important to note that there can be multiple instances of the absolute maximum or minimum value, such as on a periodic function like sine or cosine. As well, it is even more important that the interval [a, b] is closed. For the graph on the left, we would not be able to apply EVT on [-1, 1], as there are holes at x = -1 and 1! Finally, it is important to remember that the function does not have to be differentiable. Even if the function is a series of cusps or corners, these can still be absolute maxima and minima.

Let’s take a look at the Extreme Value Theorem in practice. Take the function , where we want to determine the absolute minima and maxima on the interval [-10, 10]. First, we must determine the critical points, to find any local maxima or minima. In this case, . We can determine the critical points where or DNE. We know that when the numerator is 0. Therefore, we know there are two critical points at and . As well, will be DNE when the numerator is 0, as this would result in a scenario. Therefore, there is another critical point at . Then, to determine if this critical points are at local maxima or minima, we can apply the first derivative test.

After completing the first derivative test, we find that there is a local maximum at and a local minima at , and an asymptote at (due to the scenario) which we throw out. In order to determine the absolute maxima and minima however, we must then compare the y-value of the endpoints with the y-value of the local maxima and minima. Doing so yields the following:

Based upon these results, that, for the curve on [-10,10], the absolute maximum is at (10, ) and the absolute minimum is at (-10, ).

## Guided Practice

1. Determine the number of absolute maxima and minima for the curve graphed below on [0, 5]
   1. Although we do not have a function with which to test critical points or exact y-values, we can still follow the basic principle of Extreme Value Theorem. That is, as long as a function is continuous on the given closed interval, there will be absolute extrema which are the highest/lowest points on the curve. In this example, there would be 5 absolute maxima, and 4 absolute minima. There are 5 absolute maxima, as there are 5 peaks which reach the highest y-value of 1. There are 4 absolute minima as only 4 valleys reach the lowest y-value of -1. As well, the point at is **not** an absolute minimum as there are points which are lower, and the same logic applies for the point at .
2. Determine if any maxima or minima are guaranteed per EVT for the function  on [-2, 2].
   1. Since this is a piecewise function, it is especially important to ensure that it is continuous, so that we can apply the Extreme Value Theorem. To check it’s continuity, we simplify use the three-step process outlined earlier on!
   2. We’ll first test for continuity at the possible breakpoint when . At , . The limit from the left of 0 evaluates as , and the limit from the right of 0 evaluates as . Since the limit from the left does not agree with the limit from the right, the function is not continuous at . This makes the problem much easier! If the function is not continuous, EVT does not apply, and thus we cannot guarantee any maxima or minima occur on the function.

## Lola Tries

1. Find the absolute minima and maxima for on [-3, -1].
2. Find the absolute minima and maxima for on [0, 5].
3. Find the absolute minima and maxima for on [3, 6].

Next Stop: Sri Lanka

# Welcome to Sri Lanka!

ශ්රී ලංකා වෙත ඔබව සාදරයෙන් පිළිගනිමු!